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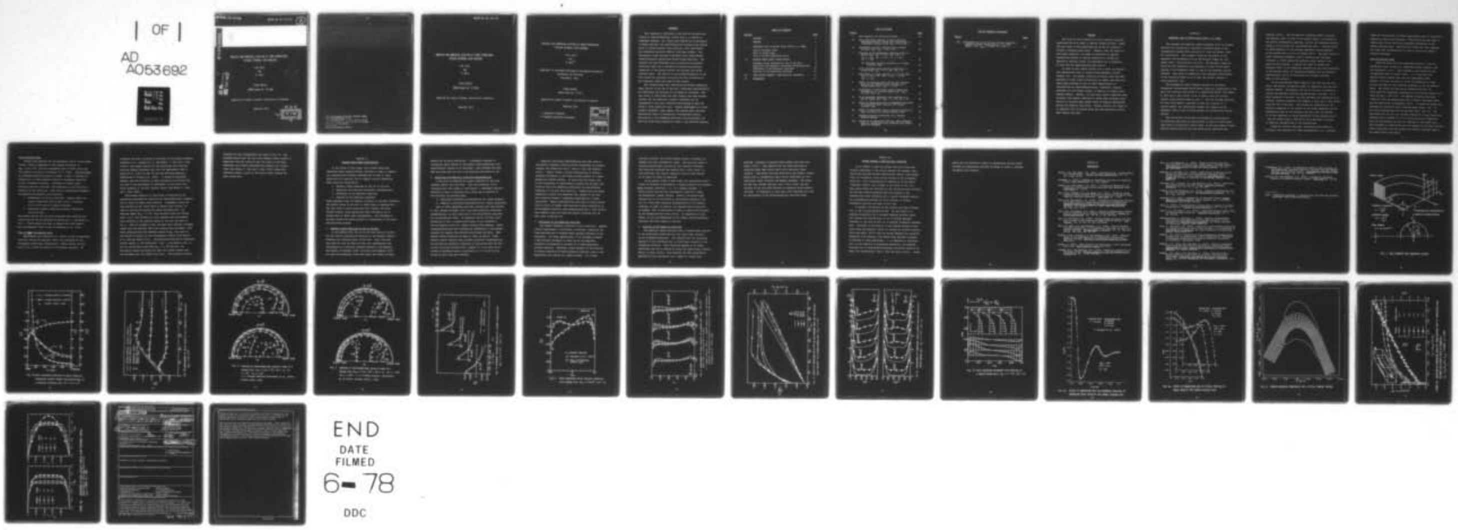

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ANALYSIS AND NUMERICAL SOLUTION OF THREE-DIMENSIONAL VISCOUS INTERNAL FLOW PROBLEMS

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See 1473 in back

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K.N. Ghia*

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U. Ghia**

Department of Aerospace Engineering and Applied Mechanics

University of Cincinnati

Cincinnati, Ohio

FINAL REPORT

AFOSR Grant No. 77-3191

February 1978

* Associate Professor

** Research Associate Professor

ABSTRACT

This research is motivated by the need for accurate prediction of three-dimensional viscous flow in a turbine or compressor passage. As a major step towards the investigation of these problems, the three-dimensional entrance flow through ducts of various regular cross sections, with longitudinal and transverse curvature effects, have been studied. The mathematical model has been formulated using time-averaged three-dimensional parabolized Navier-Stokes equations. The analysis has been developed using an existing two-equation turbulence model and is checked by obtaining satisfactory comparison of the present results for straight and curved circular pipes. The effects of the problem parameters on the flow fields are accurately evaluated and the limitations of the turbulence model have been briefly stated.

Four related areas were identified and studied separately. These consist of the law of the wall, coordinate transformations and efficiency and accuracy of the numerical algorithm. The last three of these areas have been studied with some success already, and the additional analysis developed will be implemented in the basic turbulent-flow program to make the latter a truly predictive tool. Towards development of a semi-elliptic procedure, some laminar flow results have also been obtained for ducts of rectangular cross-sections varying arbitrarily in the streamwise direction with ellipticity for the flow field being retained at least in the inviscid pressure.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF FIGURES (continued)

PREFACE

The field of fluid mechanics is built largely on special approximations which apply to defined classes of flows. Among the best known of such approximations is the one leading to Prandtl's boundary-layer theory. However, with the advent of high-speed computers, the range of simulation of applied engineering problems is rapidly advancing to include more physically realistic flows, as researchers try to gradually do away with the approximations used earlier.

In order to gain useful insight into a class of internal-flow applications such as turbine blade passages, aircraft intakes, etc., the present study was initiated using the model problems of flows inside curved ducts of simple cross-sections. These model problems as well as their applications are characterized by three-dimensionality, turbulence, pressure gradients and centrifugal forces, in addition to several other complex effects. Section I of this final report summarizes some turbulent flow results obtained for these model problems, while Section II outlines some related areas of research encountered during the course of this study. Finally, Section III refers to an important area of further study and some preliminary effort made towards that goal.

SECTION I

TURBULENT FLOW IN CURVED DUCTS USING A k-ε MODEL

The analysis and numerical method developed serve the primary objective of introducing an available turbulence model in the corresponding laminar-flow analysis of K. Ghia and Sokhey (1976) for curved ducts. This enables a study of the effect of the important flow parameters such as the Reynolds number Re, the aspect ratio γ and curvature ratio R/D. The secondary objective served by this research is the evaluation of some of the short-comings of the turbulence model in view of the numerical results obtained. This leads to suggestions about possible modifications of the turbulence model so as to make the present analysis a truly predictive tool.

The mathematical model employed consists of the three-dimensional time-averaged Navier-Stokes equations, parabolized in the main-flow direction and written in a generalized manner [see Sokhey, K. Ghia and U. Ghia (1978)] so as to easily treat the various regular cross sections shown in Fig. 1. The use of these equations for duct flows makes it possible to take into account, in a unified manner, the interactions between the primary and the secondary flows as well as those between the inviscid and viscous regions.

The statistical correlations introduced by time-averaging the governing equations are approximated in terms of known quantities using a second-order closure model, as the more complex stress-equation models have not yet been sufficiently developed [see

Reynolds (1974)]. The two-equation turbulence model of Launder and Spalding (1974) is used in the present study. It consists of two partial differential equations, one for the turbulence kinetic energy k and the other for its dissipation rate ε. The derivation of these equations in the toroidal coordinate system employed in the present work is given by Sokhey (1977). The local values of these turbulence parameters help to determine the turbulent viscosity ν_t which relates the effective shear stress to the mean-velocity gradient. Although the model of Launder and Spalding (1974) is not completely adequate for the flows considered in the present study, its simplicity, together with its possible modifications, led to its selection as a suitable starting model in the analysis.

In the low-Reynolds number region of the flow near the duct walls, the present two-equation model of turbulence uses the wall-function method. Accordingly, the law of the wall is invoked to provide the necessary boundary conditions near a surface. At a point adjacent to the wall, the flow variables are determined by a suitable iterative procedure [see Sokhey, K. Ghia and U. Ghia (1978)] using the law of the wall. This avoids the integration of the governing equations in the region of very steep gradients and the accompanying need for an extremely fine grid near the surface. It is also important to choose consistent initial profiles for k, ε and the length scale ℓ; failure to do so was found to result in numerical instabilities in the calculations.

Using the analysis developed, some preliminary numerical solutions were generated for some configurations, e.g., straight pipes and curved pipes, for which experimental data or analytical asymptotic results are available. These solutions were used to enable selection of the finite-difference parameters of the problem appropriately. Detailed flow fields were then computed for these test configurations in order to verify the general analysis developed.

Flow in Straight Pipes

Figure 2a shows the fully developed profiles of the non-dimensional axial velocity w, turbulence kinetic energy k, and the dissipation rate ε. Also included in the figure are the corresponding data of Laufer (1951). It is seen that the present results compare well with the experimental data. However, the axial locations at which the comparison is made are not the same for the numerical solutions as for the measurements. The reason for this can be inferred from Fig. 2b which shows the streamwise variations of w at various radial locations for $Re_D = 3.88 \times 10^5$. The present solutions show good comparison with the experimental data of Barbin and Jones (1963) and the predictions of Stephenson (1975). It should be noted that, although the centerline axial velocity peaks around $z/D \simeq 32$, the fully developed flow is not attained until $z/D \simeq 55$. The nonuniform grid of (42 x 11) points referred to in this figure was obtained from a uniform (41 x 11) grid by placing an additional point at the mid-point of the cell adjacent to the radial wall. This procedure of subdividing the grid adjacent to the wall provides better resolution near the surface and leads to improved results but needs further evaluation.

Flow in Curved Pipes

Results were obtained for the developing flow in curved pipes. Figure 3 shows a comparison of the present solutions, at $Re_D = 2.36 \times 10^5$, with the experimental data of Rowe (1970) and the numerical solutions of Patankar et al. (1975). The development of streamwise-velocity contours is shown in Figs. 3a through 3d, whereas Fig. 3e shows typical profiles of secondary velocities at four streamwise locations. Although all results agree favorably near the outer wall, the comparison is not very satisfactory near the inner wall. This lack of satisfactory agreement is presently attributed to the following:

1. Differences in initial conditions. Details about the initial conditions used by Rowe (1970) or Patankar et al. (1975) are not readily available.
2. Inaccurate boundary conditions entering through the law of the wall.

The present results for the fully developed axial velocity profiles along the two mid-surfaces AA and BB (Fig. 1) are shown in Fig. 4. These results are seen to compare well with those of Mori and Nakayama (1965) as well as Patankar et al. (1975).

Flow in Curved Rectangular Ducts

Experimental data [Howard et al. (1975)] as well as numerical solutions [Pratap and Spalding (1975)] are available for one rectangular curved-duct configuration. These results are comared in Fig. 5 with the results of the present analysis. By retaining the local variation of curvature in the present parabolic procedure [i.e., setting NS = 1; see Sokhey, K. Ghia and U. Ghia (1977)], the present results for the developing axial velocity profiles compare reasonably well with the experimental data of Howard et al. (1975) as well as with the partially-parabolic predictions of Pratap et al. (1975). It should be mentioned, however, that the inclusion of local variations of curvature did not lead to the same degree of improvement in the results for static pressure in the duct entrance region [see Sokhey, K. Ghia and U. Ghia (1978)].

The primary objective of this study is fulfilled by the formulation developed for analyzing the three-dimensional turbulent flow in ducts of simple cross sections. A parametric study was then carried out for curved ducts of square cross section, in order to provide new results for these configurations. For Reynolds number $Re_D = 5 \times 10^4$, with curvature ratio R/D ranging from 5 to 50, the variation of axial pressure drop and maximum axial velocity in the entrance region of the duct are shown in Fig. 6. With an increase in R/D, the peak axial velocity increases except near the entrance, while the pressure drop decreases. With these configurations with moderate values of Re_D, the trend of the results is similar to the corresponding laminar flow results of K. Ghia and Sokhey (1977). The developing profiles of turbulence kinetic energy, k, and dissipation rate, ε, are shown in Fig. 7a for the curved square duct with $Re_D = 5 \times 10^4$ and R/D = 20. The kinetic energy increases very near the outside concave wall and decreases near the inside convex wall. The secondary-velocity profiles for this configuration are shown in Fig. 7b. The secondary motion near the duct walls changes rather rapidly in comparison with the variation near the center of the duct. Comparison with typical secondary-flow patterns for laminar flows [see Sokhey, K. Ghia and U. Ghia (1978)] shows that turbulence causes a shift in the vortex center towards the inner convex wall.

SECTION II

RELATED AREAS UNDER INVESTIGATION

In the course of this study, four related areas were identified which required further attention in order to improve the turbulent-flow analysis developed and to make it truly predictive for turbulent flow in complex internal geometries. These areas are listed below:

1. Boundary values generated by the law of the wall.
2. Analytical and numerical coordinate transformations.
3. Efficiency of the numerical algorithm.
4. Accuracy of the numerical algorithm.

These represent areas of research currently in progress; therefore, the present status of the work in these areas is reported here. The last three of these areas have been studied, with some success already, using appropriate model problems so as to analyze each of these areas independently. The knowledge as and when gained from these analyses is incorporated in the basic turbulent-flow program.

1. Boundary Values Generated by Law of the Wall

In its present form, the law of the wall does not account for the viscous interactions occurring in the boundary regions formed by the adjoining walls of the duct. Also, the basic form used for this law does not take into consideration the longitudinal or transverse curvature present in the geometry. The studies of Bradshaw (1973) and Wilcox and Chambers (1976) for some two-dimensional flows have shown that these curvature effects can be quite significant. A systematic approach to incorporate these effects in the present three-dimensional flow configurations remains to be developed. Some attempts have been made but these have not led to any definite conclusions so far.

2. Analytical and Numerical Coordinate-Transformations

A fine grid is required for proper resolution of the high-gradient regions near the walls. With the constraint of not increasing the total number of grid points, a reasonable approach consists of using a nonuniform grid, which may be achieved in the following two possible ways:

a. Analytical coordinate-transformations for simple geometry.

b. Numerical coordinate-transformations for complex geometries.

Both of these approaches have been analyzed with some success. K. Ghia, Hankey and Hodge (1977) have used analytical coordinate transformations in their solutions of the Navier-Stokes equations for two-dimensional flows. An important feature of these transformations is that it permits grid points to be clustered at the two opposite walls of a simple internal-flow geometry. These transformations have been successfully implemented in the present analysis for laminar flow through curved pipes and ducts. Some typical results obtained are shown in Figs. 8a and 8b. These results clearly indicate that suitable analytical transformations can provide the necessary clustering of grid points near the wall so that solutions obtained with a uniform (21 x 21) grid can be virtually reproduced using a nonuniform (11 x 11) grid for the curved circular pipe flow analyzed.

Numerical coordinate-transformations have been used to successfully generate surface-oriented coordinates for complex internal flow geometries [see U. Ghia, K. Ghia and Studerus (1976)]. Figure 9 shows a typical distribution of surface-oriented coordinates in the blade-to-blade surface of a three-dimensional cascade passage. As seen from this figure, one of the coordinates is approximately aligned with the main flow direction while the remaining coordinates are approximately orthogonal to the main flow direction. Parabolizing the three-dimensional viscous flow equations with respect to the main flow direction becomes a reasonable approximation in these coordinates. These coordinates have been used to study laminar flows in ducts of simple cross sections varying arbitrarily in the streamwise direction [see Sathyanarayana and U. Ghia (1978)]. Some comments about the resulting typical solutions will be made later in Section III.

3. Efficiency of the Numerical Algorithm

The present numerical algorithm is quite efficient. However, further improvements in efficiency are desired, especially for those applications of interest where upstream influence is not negligible. For these applications, it is intended to employ a semi-elliptic procedure in which one of the dependent variables is determined by solution of a three-dimensional elliptic equation. Therefore, direct methods for the solution of Poisson equations are being presently examined, using a two-dimensional flow problem as a model problem. For a model

Dirichlet problem, the direct Poisson solvers of Hockney and Buneman have been successfully coded. Care has been taken to maintain second-order accuracy for the numerical solutions. The resulting algorithm [see Osswald and K. Ghia (1978)] is very efficient; the typical computing time has been reduced by at least one order of magnitude.

In the present study of duct flows, the cross-plane distribution of pressure is governed by a Poisson equation with Neumann-type boundary conditions, i.e., by a Neumann problem. A direct solver has also been successfully coded for a model two-dimensional Neumann problem. However, the second-order accuracy had to be sacrificed at the boundary because of the use of a first-order accurate representation of the boundary condition in order to maintain the required matrix structure. At present, work is continuing in order to improve the accuracy of the Neumann-problem direct solver. On completion of this effort, it will be implemented in the present three-dimensional turbulent duct-flow numerical analysis.

4. Accuracy of the Numerical Algorithm

The numerical method presently used is second-order accurate in the cross-plane directions but only first-order accurate in the streamwise direction. This implies that the continuity equation is also satisfied only to first-order accuracy in the streamwise direction. This limitation appears to be rather undesirable and can be removed by using the higher-order accurate theory of cubic splines. This approach has been successfully employed by Rubin and Khosla for a number of viscous-flow problems, including a turbulent-flow problem [see Rubin and Khosla (1977)]. This approach has the added advantage of requiring fewer mesh points while still providing higher accuracy. Some effort has been directed by the present investigators also in implementing cubic-spline algorithms for two-dimensional model Dirichlet and Neumann problems. The effort has had some success [see Shin and K. Ghia (1978)] and the accuracy of the solutions obtained is presently being formally verified. On completion of this work, this algorithm will also be incorporated in the three-dimensional duct-flow study.

SECTION III

EFFORT TOWARDS A SEMI-ELLIPTIC PROCEDURE

In an attempt to make the present duct-flow study more directly applicable to the actual cascade flow passage, some effort was directed towards analyzing the laminar flow through ducts of rectangular cross section varying in the streamwise direction. This configuration is referred to as the planar 'Joukowski Venturi' and is shown in the inset in Fig. 10a. This figure shows the variation of the centerline axial velocity and the mean viscous-pressure drop with respect to the dimensionless axial coordinate for a typical Joukowski Venturi. The corresponding profiles of axial velocity at various streamwise locations are shown in Fig. 10b.

At present, ellipticity of the solution has been retained only in the inviscid pressure field. The next step is to consider ellipticity for a variable computed directly within the viscous flow calculations, so as to include upstream influence in the flow as well as to consider streamwise reversed flow. Inclusion of ellipticity in the complete pressure field [see Pratap and Spalding (1976)] or through an inviscid velocity field [see Dodge (1977)] have not proved adequate for analyzing streamwise flow reversal, even though some upstream influence is retained in these approaches. It is important to recognize that, in the presence of streamwise separation, the upstream influence due to displacement-thickness interaction needs to be taken into consideration [see U. Ghia and Davis (1974)]. Future effort will be directed to obtain an appropriate analysis which includes the appropriate upstream influence in order to consider streamwise flow reversal.

SECTION IV

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Note: Superscript asterisk (*) denotes that the work was partially performed under AFOSR Sponsorship.

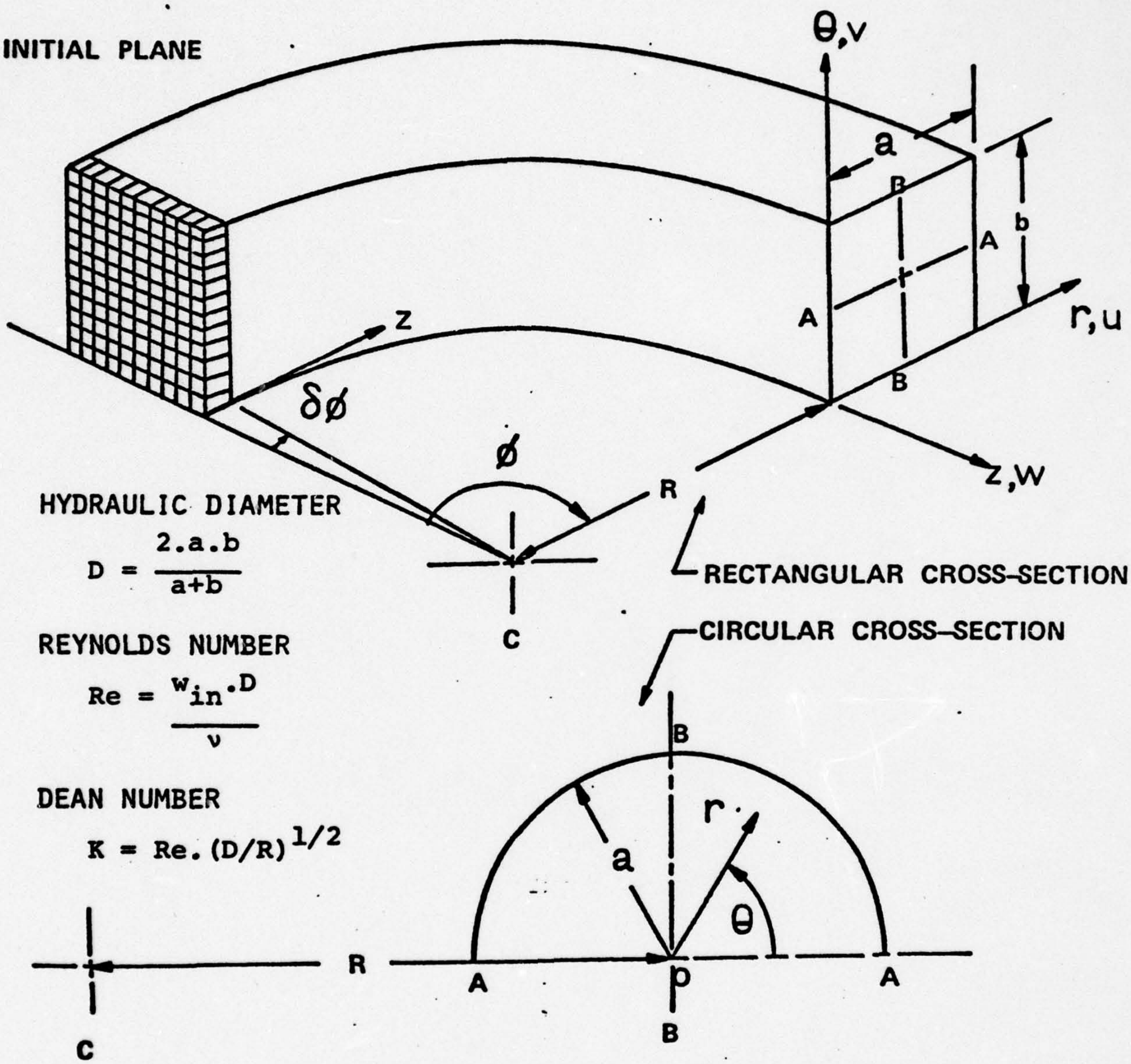


FIG. 1 DUCT GEOMETRY AND COORDINATE SYSTEM

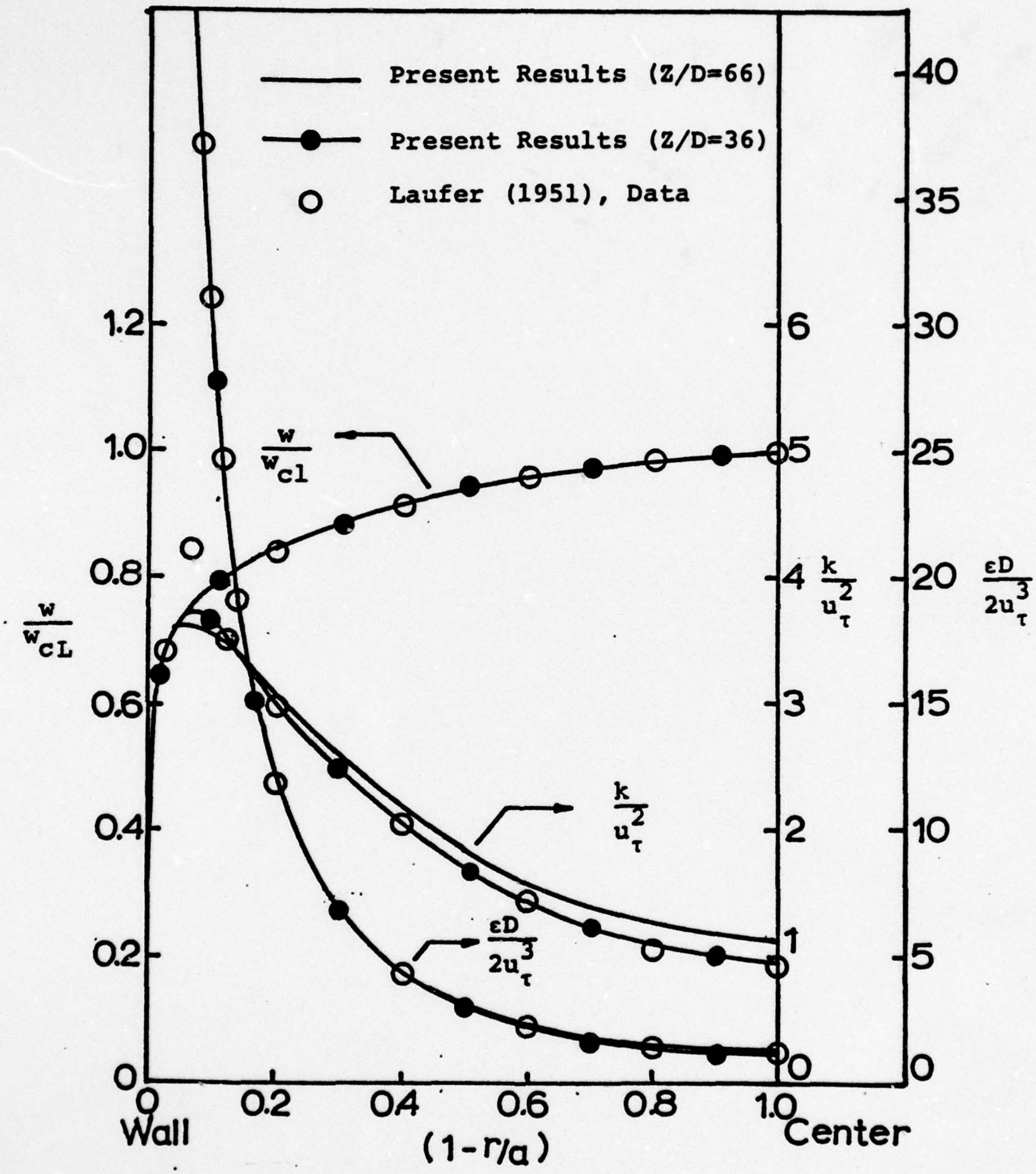


FIG. 2A FULLY DEVELOPED PROFILES OF AXIAL VELOCITY, TURBULENCE KINETIC ENERGY AND DISSIPATION IN A STRAIGHT CIRCULAR PIPE, Re = 5.0×10^5

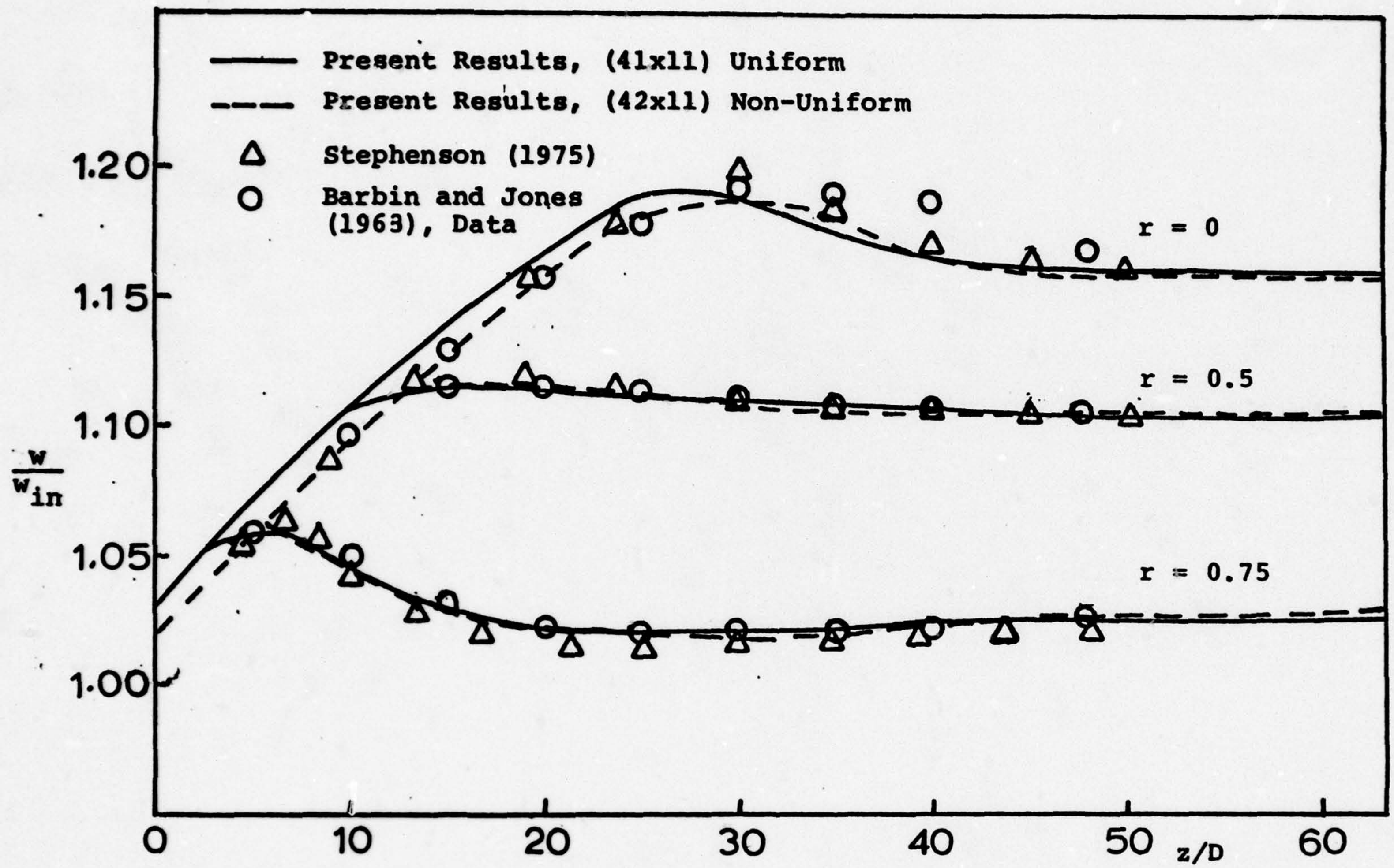


FIG. 2B DEVELOPMENT OF AXIAL VELOCITY IN A STRAIGHT CIRCULAR PIPE,
$Re_D = 3.88 \times 10^5$

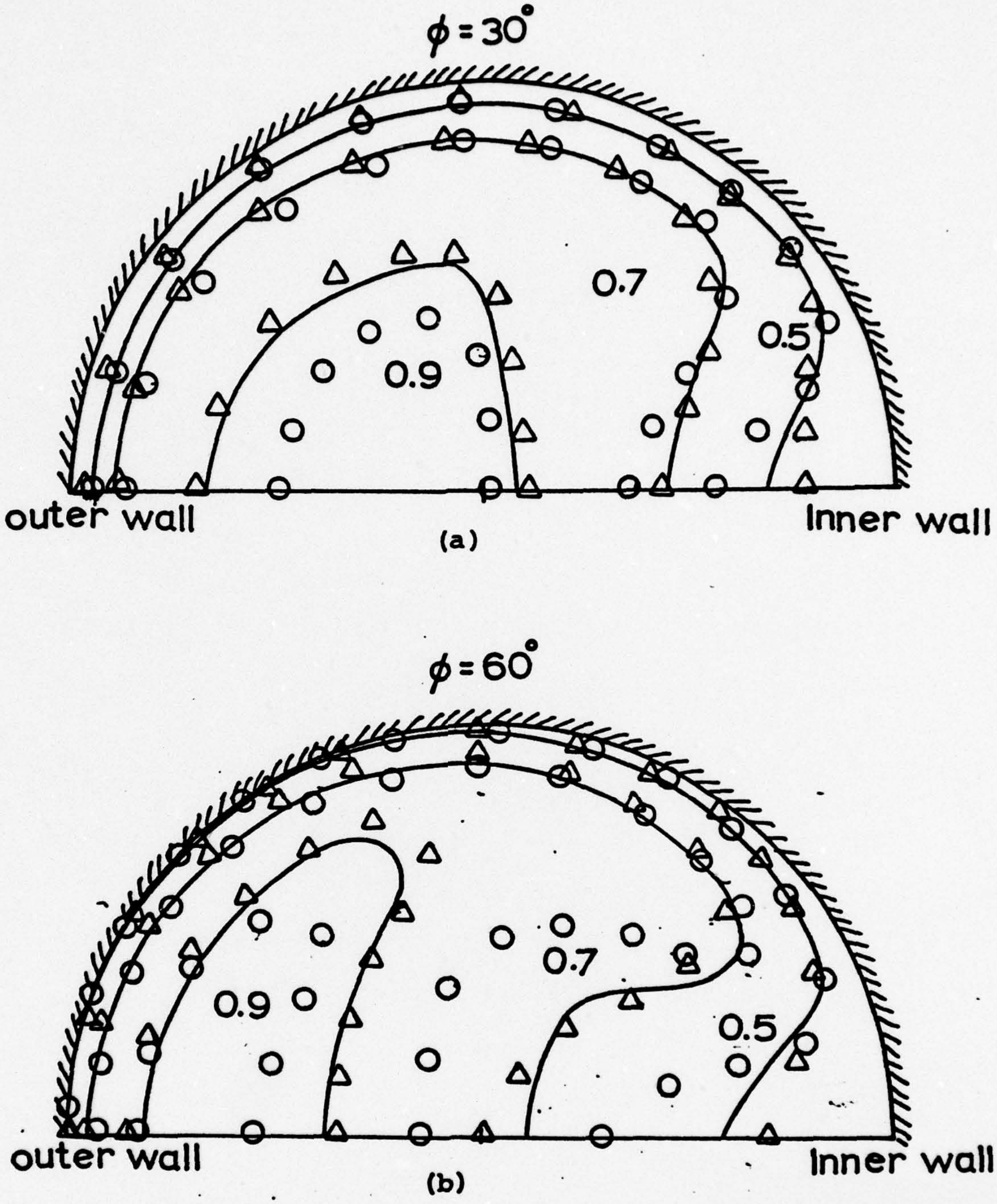


FIG. 3 CONTOURS OF NON-DIMENSIONAL VELOCITY HEAD IN A CURVED PIPE, $Re_D = 2.36 \times 10^5$, R/D = 12 (a) $\phi = 30^\circ$ (b) $\phi = 60^\circ$

——— **Present Results; △ Patankar et al. (1975);**

○ **Rowe (1970), Data**

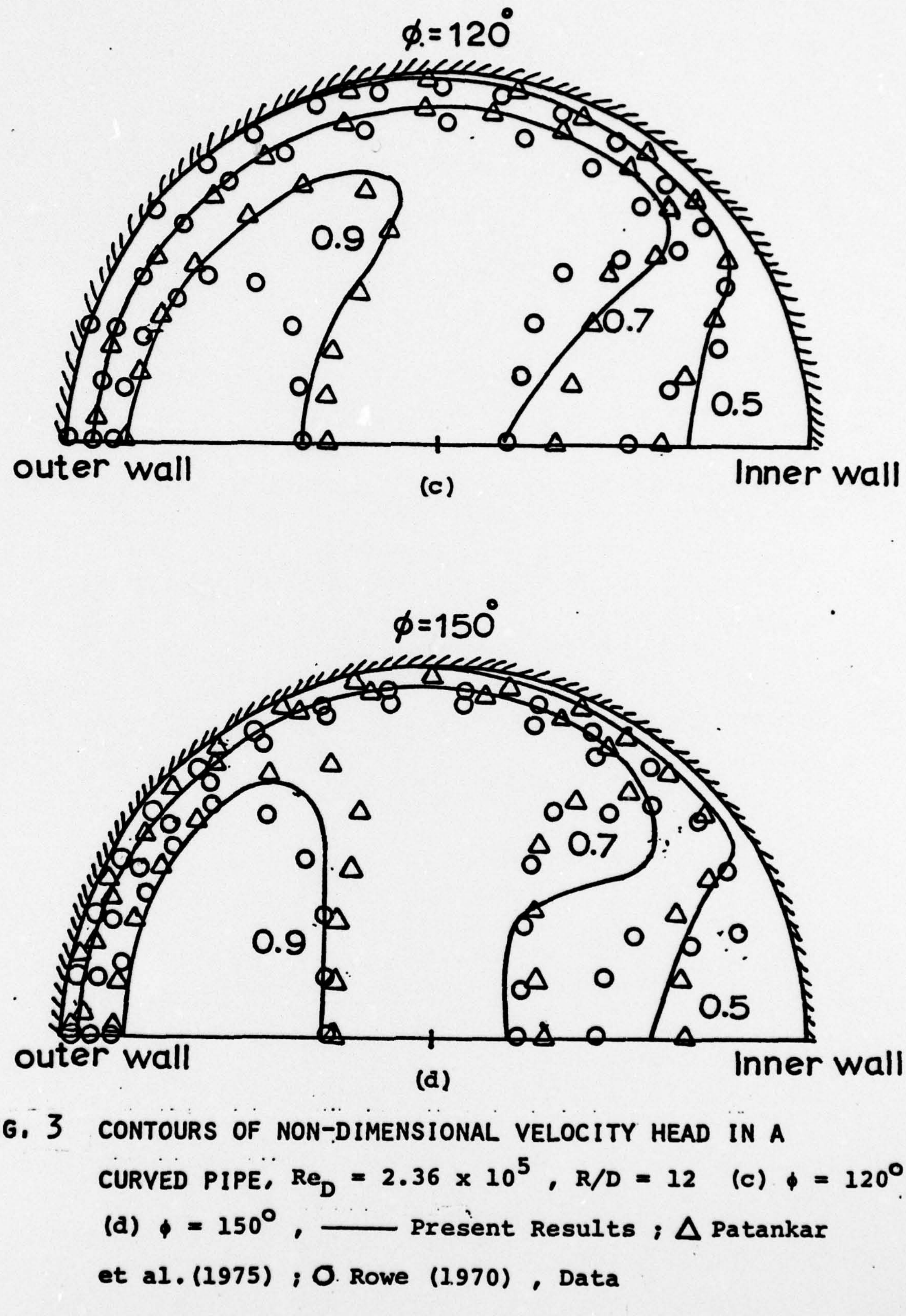
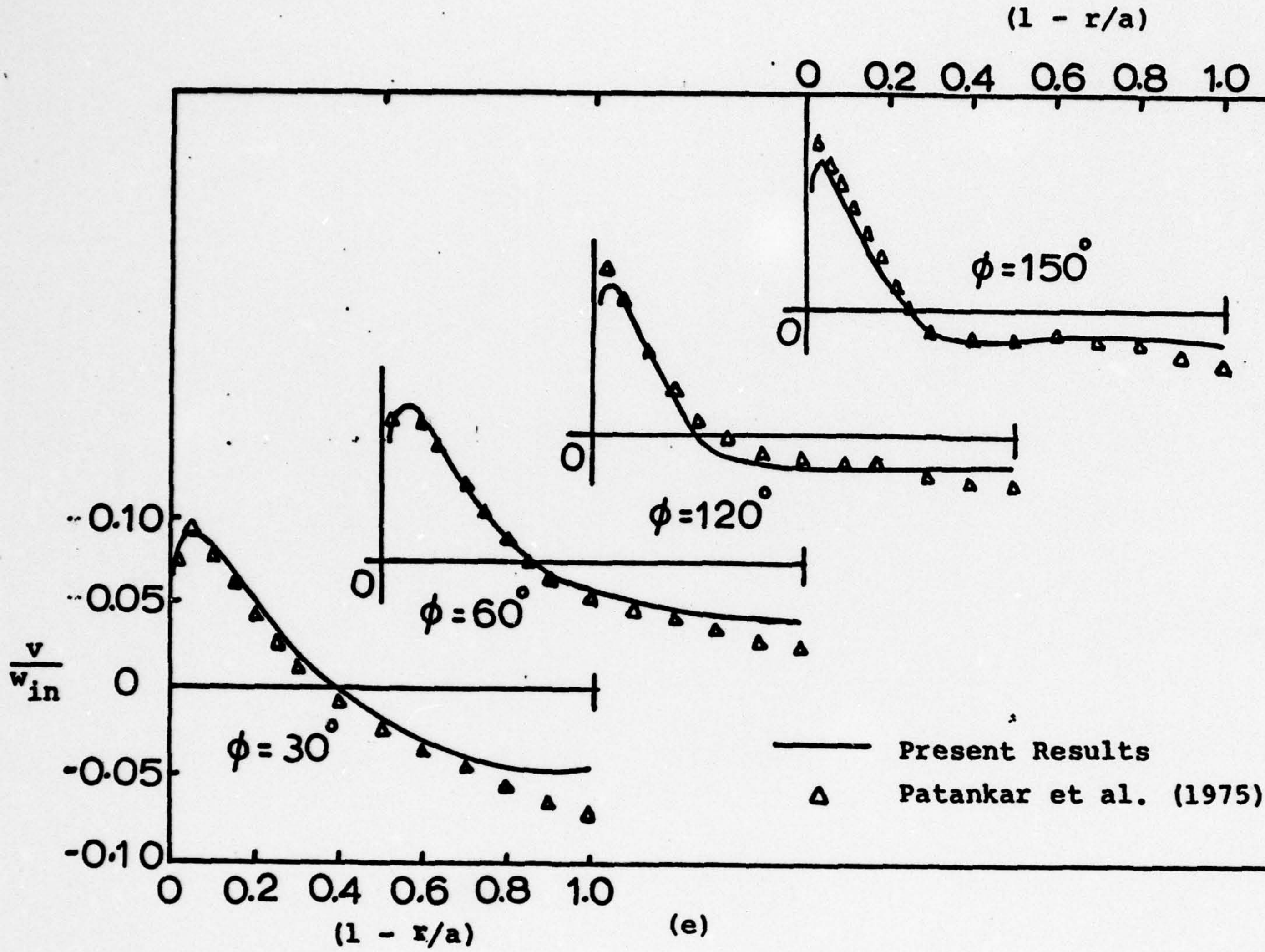


FIG. 3 CONTOURS OF NON-DIMENSIONAL VELOCITY HEAD IN A CURVED PIPE, $Re_D = 2.36 \times 10^5$, R/D = 12 (c) $\phi = 120^\circ$ (d) $\phi = 150^\circ$, ——— Present Results ; △ Patankar et al.(1975) ; ○ Rowe (1970) , Data



(e)

FIG. 3e SECONDARY VELOCITY PROFILES IN A CURVED CIRCULAR PIPE.

$Re_D = 2.36 \times 10^5$, R/D = 12

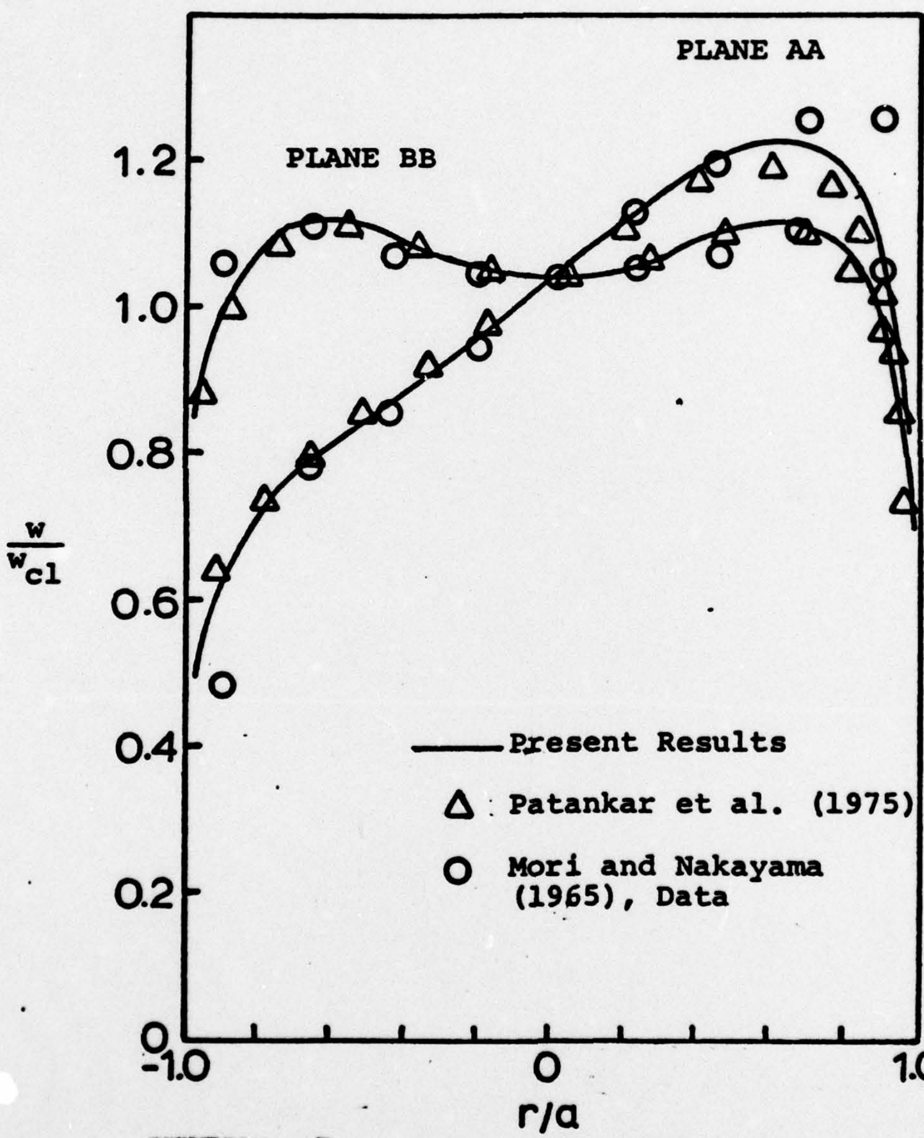


FIG. 4 FULLY DEVELOPED AXIAL VELOCITY PROFILES IN A CURVED PIPE, $Re_D = 2.5\times10^4$, $R/D = 20$

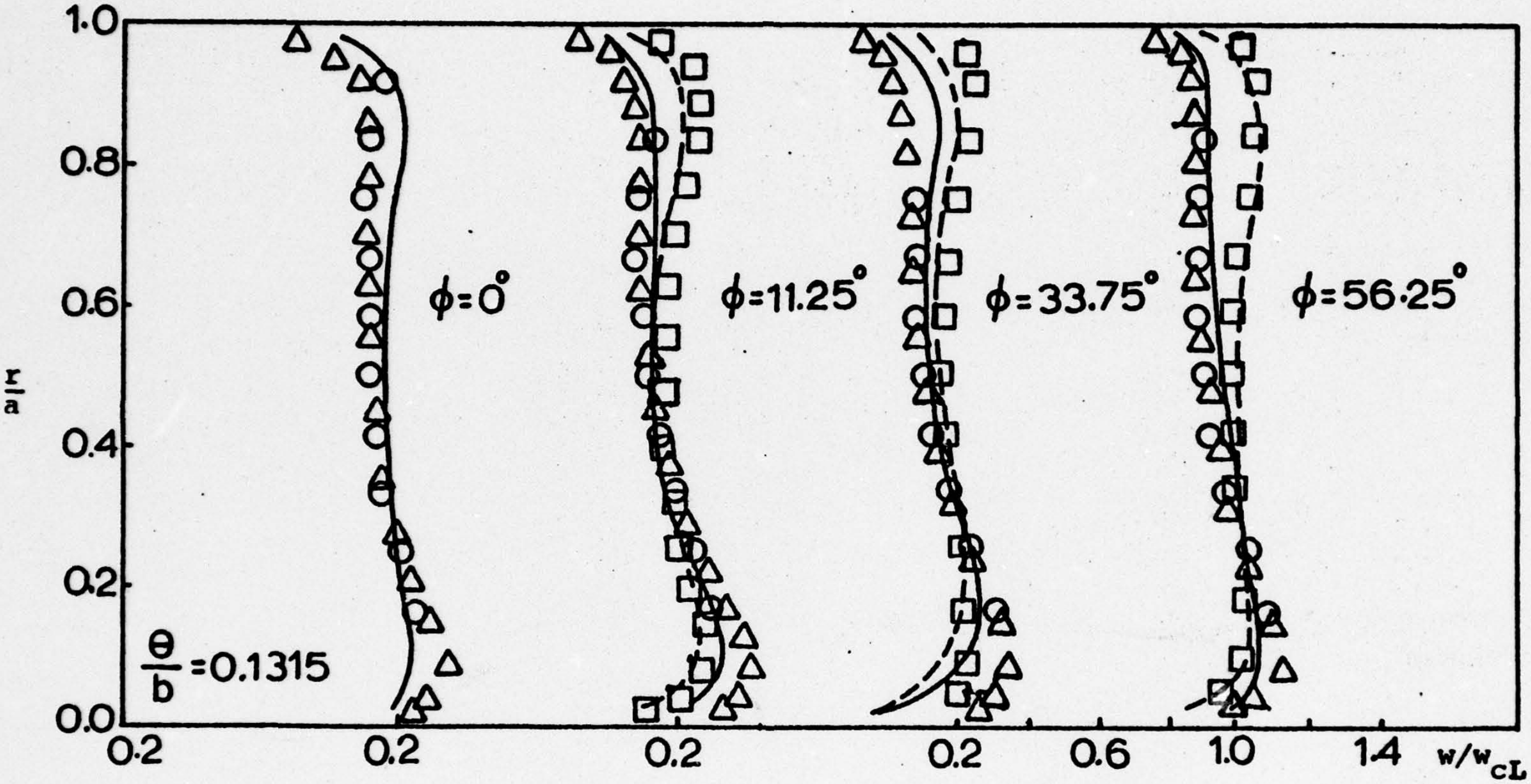


FIG. 5 DEVELOPMENT OF AXIAL VELOCITY IN A CURVED DUCT, $Re_D = .706 \times 10^6$, $R/D = 3.92$, $\gamma = 0.25$ at $\theta/b = 0.1315$;

—— Present Results (NS=1); --- Present Results (NS=0); □ "Parabolic" solution, △ "partially parabolic" solution, Pratap and Spalding (1975), ○ Howard et al. (1975), Data

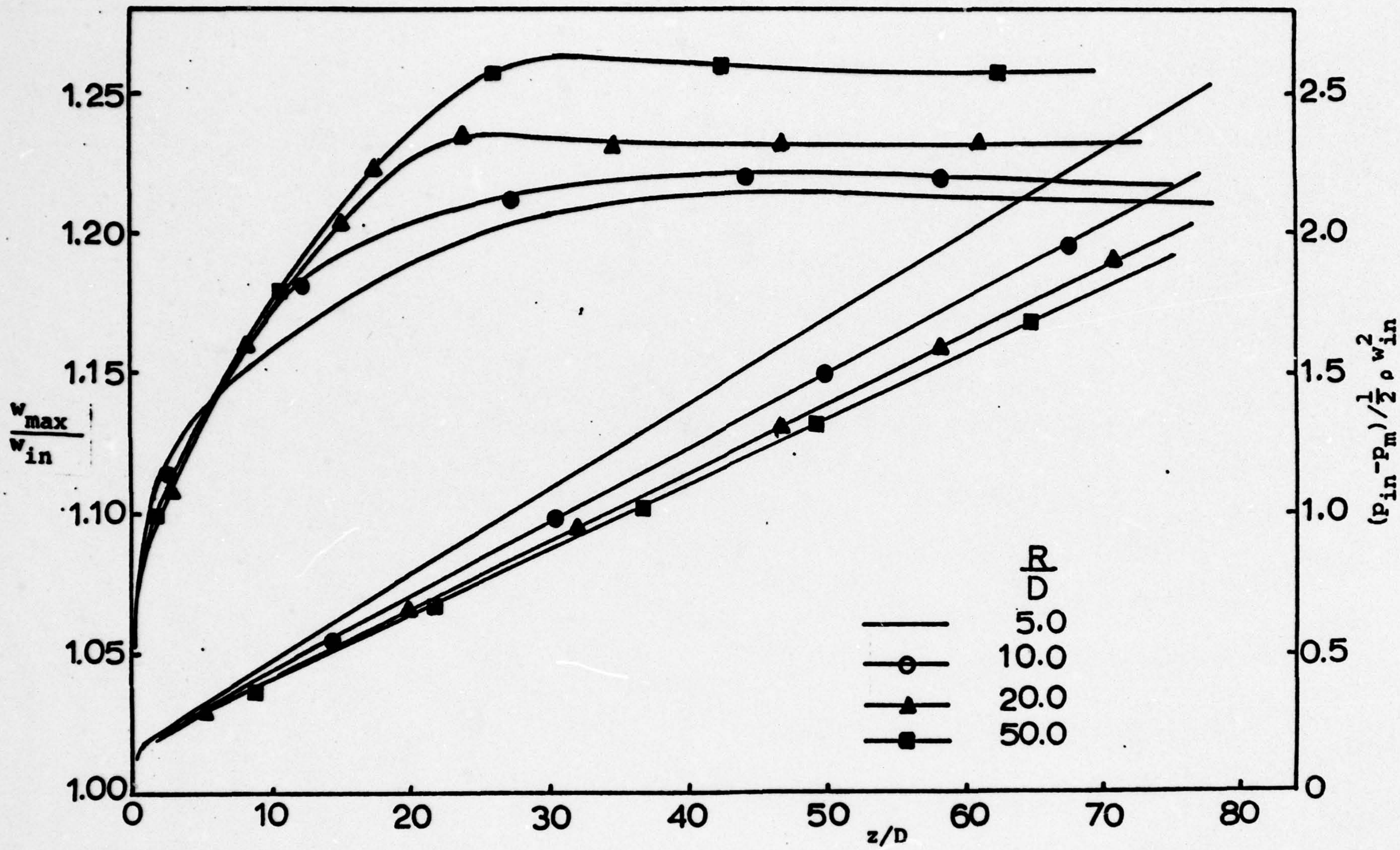


FIG. 6 EFFECT OF CURVATURE RATIO R/D ON THE PRESSURE DROP AND THE MAXIMUM AXIAL VELOCITY IN A CURVED SQUARE DUCT, $Re_D = 5 \times 10^4$

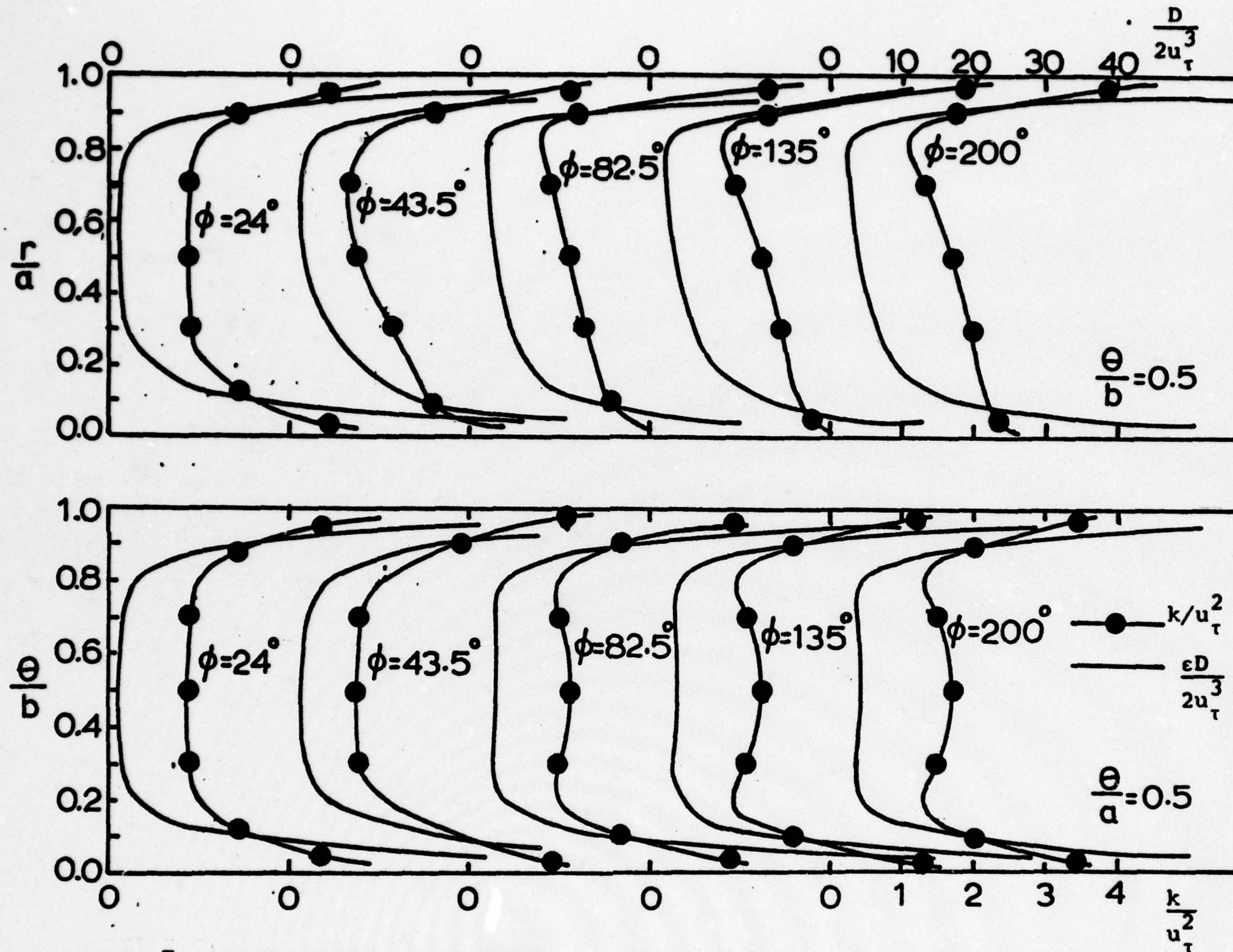


FIG. 7A DEVELOPMENT OF TURBULENCE KINETIC ENERGY AND DISSIPATION IN A CURVED SQUARE DUCT, $Re_D = 5 \times 10^4$, R/D = 20

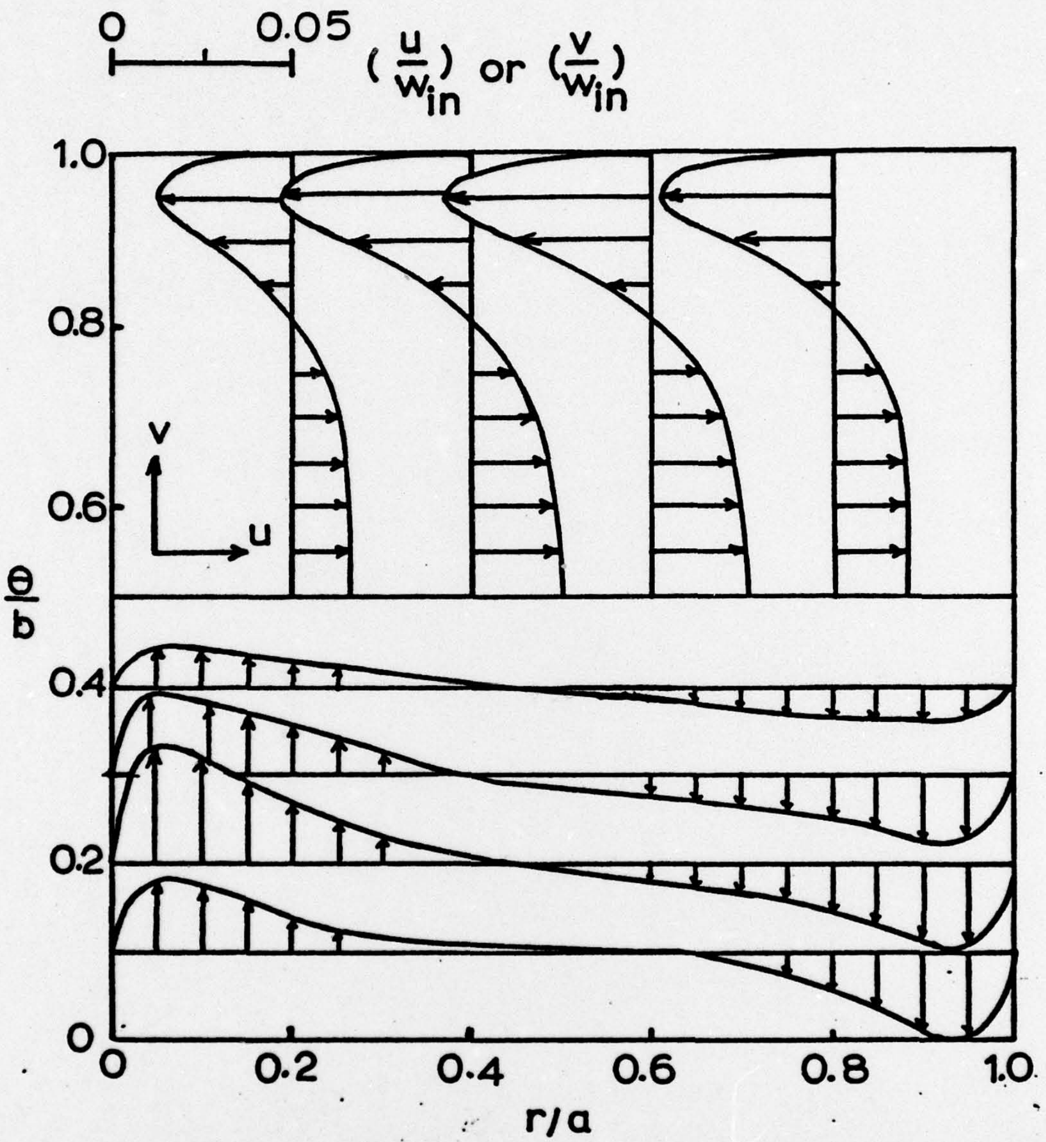


FIG. 7B FULLY DEVELOPED SECONDARY FLOW PROFILES IN A CURVED SQUARE DUCT, $Re_D = 5 \times 10^4$, R/D = 20

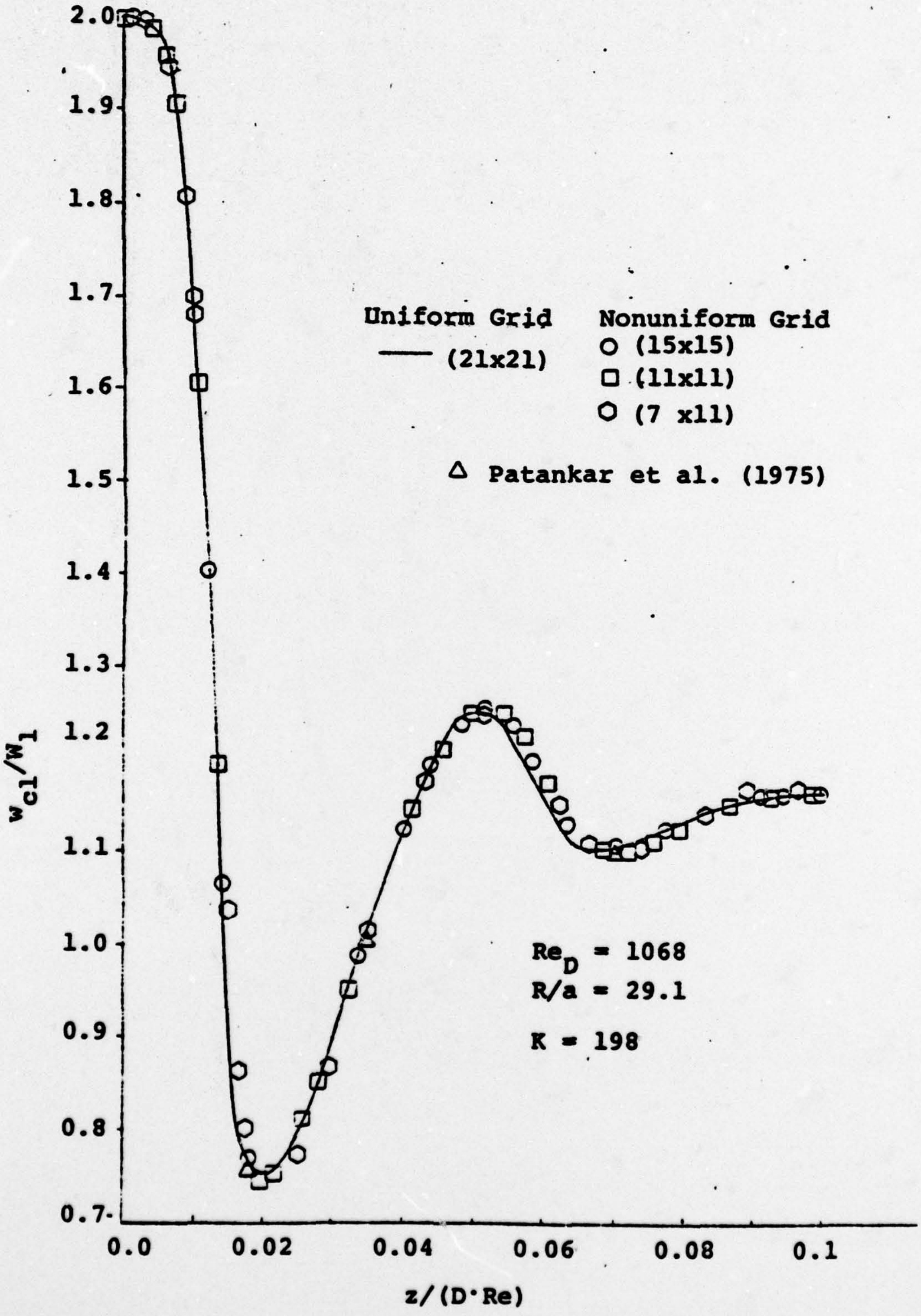


FIG. 8A. EFFECT OF NONUNIFORM GRID ON STREAMWISE VARIATION OF CENTERLINE AXIAL VELOCITY FOR CURVED CIRCULAR PIPE.

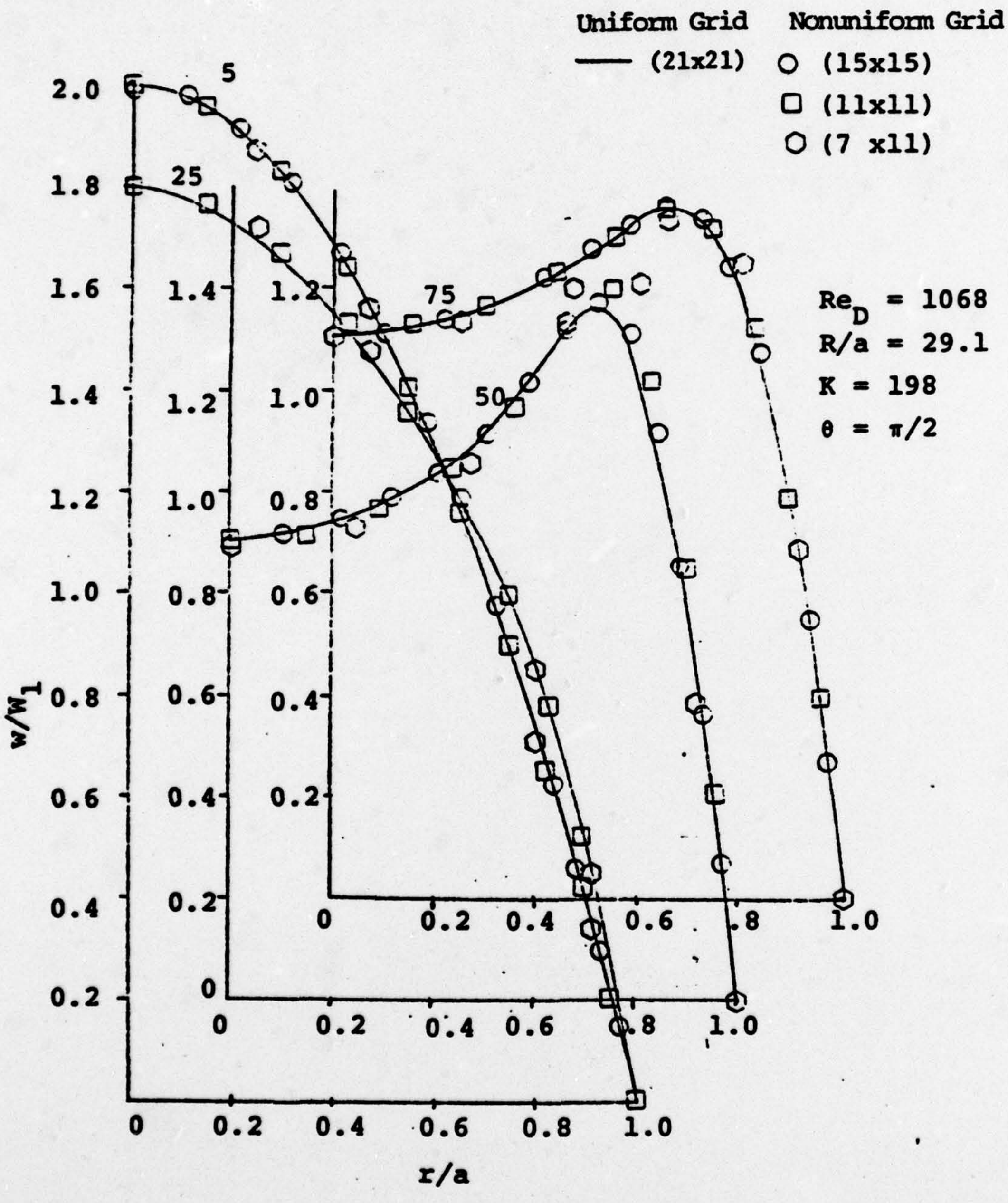


FIG. 8B. EFFECT OF NONUNIFORM GRID ON TYPICAL PROFILES OF AXIAL VELOCITY FOR CURVED CIRCULAR PIPE.

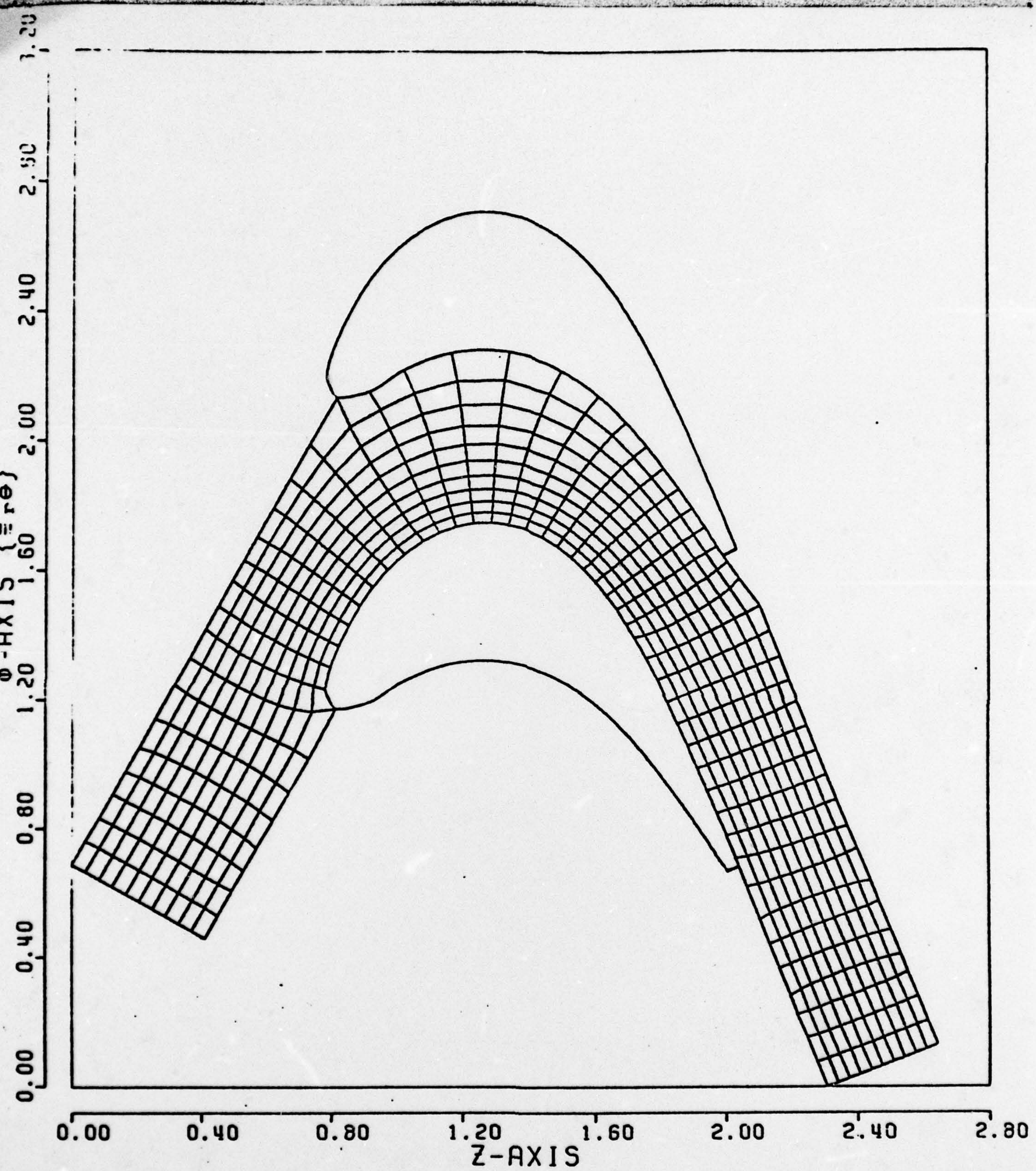


FIG. 9. SURFACE-ORIENTED COORDINATES FOR A TYPICAL TURBINE CASCADE.

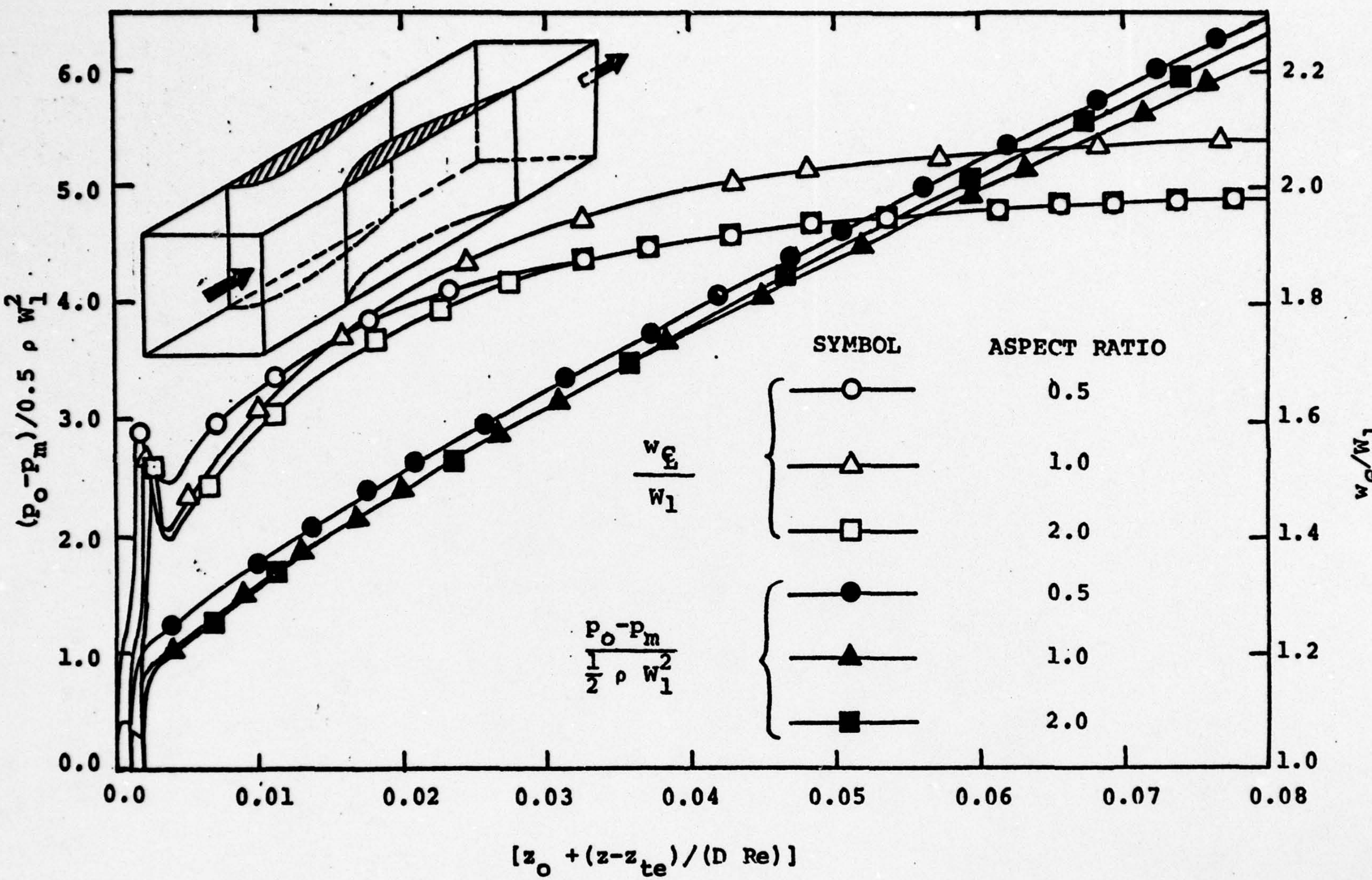


FIGURE 10A. VARIATION OF CENTERLINE VELOCITY, MEAN PRESSURE DROP VS. NORMALIZED AXIAL DISTANCE WITH ASPECT RATIO AS PARAMETER.

$Z_0 = (Z_{te}/D_{ref}Re)$, $D_{ref} = 0.41867$, $Re = 1000$

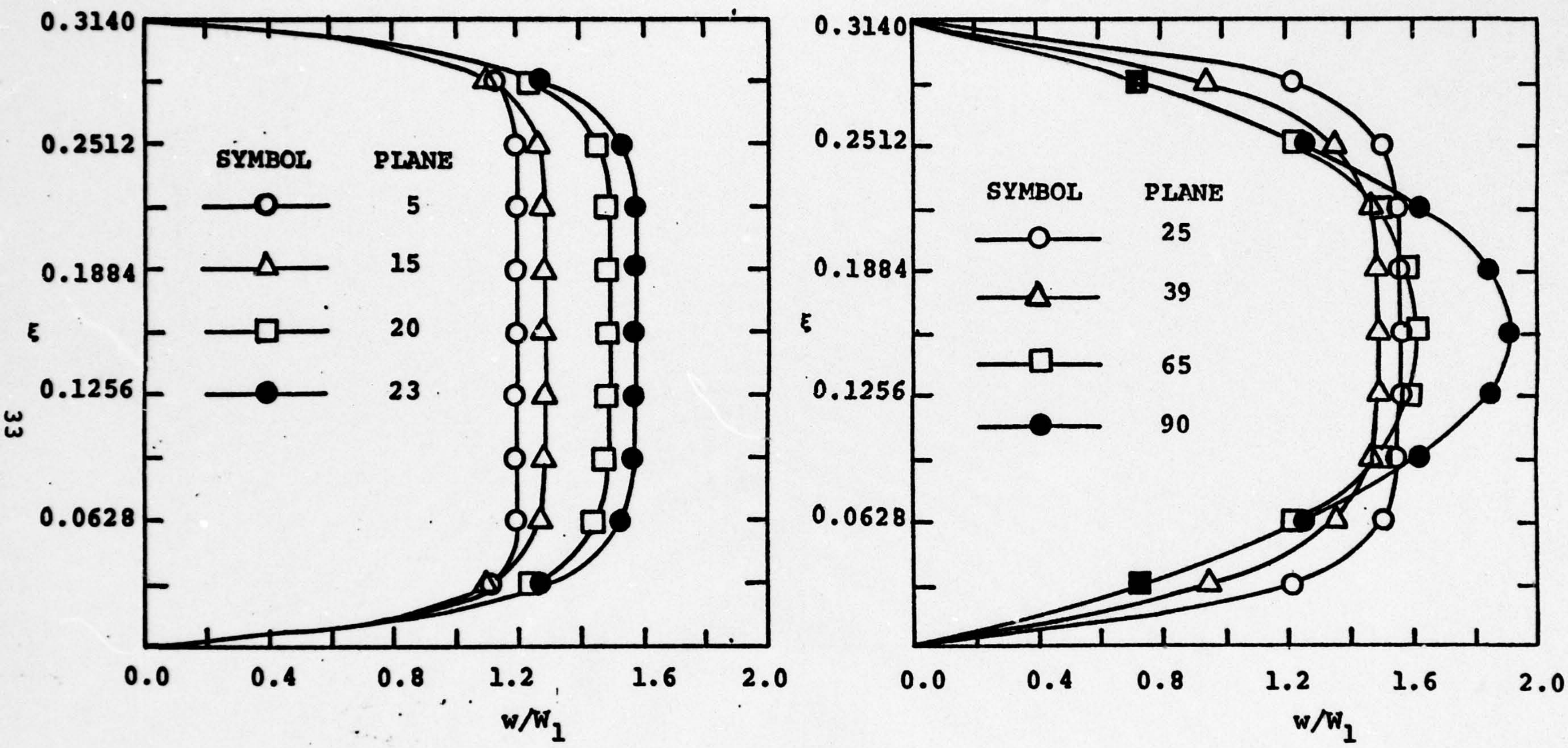


FIGURE 10B. DEVELOPMENT OF AXIAL VELOCITY PROFILE ALONG MIDCHANNEL SURFACE.
ASPECT RATIO = 0.5, AREA RATIO = 0.8
t_{max} = 0.0628, RE = 1000

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LAMINAR AND TURBULENT	k-ε TURBULENCE MODEL
INCOMPRESSIBLE	IMPLICIT NUMERICAL SOLUTIONS
CIRCULAR AND RECTANGULAR CURVED DUCTS	DIRECT SOLVERS
PARABOLIZED NAVIER-STOKES EQUATIONS	COUPLED SPLINE ALGORITHM

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This research is motivated by the need for accurate prediction of three-dimensional viscous flow in a turbine or compressor passage. As a major step towards the investigation of these problems, the three-dimensional entrance flow through ducts of various regular cross-sections, with longitudinal and transverse curvature effects, have been studied. The mathematical model has been formulated using time-averaged three-dimensional parabolized Navier-Stokes equations. The analysis has been developed using an existing two-equation

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turbulence model and is checked by obtaining satisfactory comparison of the present results for straight and curved circular pipes. The effects of the problem parameters on the flow fields are accurately evaluated and the limitations of the turbulence model have been briefly stated.

Four related areas were identified and studied separately. These consist of the law of the wall, coordinate transformations and efficiency and accuracy of the numerical algorithm. The last three of these areas have been studied with some success already, and the additional analysis developed will be implemented in the basic turbulent-flow program to make the latter a truly predictive tool. Towards development of a semi-elliptic procedure, some laminar flow results have also been obtained for ducts of rectangular cross-sections varying arbitrarily in the streamwise direction with ellipticity for the flow field being retained at least in the inviscid pressure.